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ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)
B.E. / B. Tech (Full Time) - END SEMESTER EXAMINATIONS, APRIL/MAY 2024
Common to all branches (Except CSE)
II Semester
MA3251- Ordinary Differential Equations & Transform Techniques
(Regulation 2023)

Time: 3hrs

Max. Marks: 100

CO 1	Solve higher order ordinary differential equations which arise in engineering applications.
CO 2	Apply Laplace transform techniques in solving linear differential equations.
CO 3	Apply Fourier series techniques in engineering applications.
CO 4	Understand the Fourier transforms techniques in solving engineering problems.
CO 5	Understand the Z-transforms techniques in solving difference equations.

BL – Bloom's Taxonomy Levels
(L1 - Remembering, L2 - Understanding, L3 - Applying, L4 - Analysing, L5 - Evaluating, L6 - Creating)

PART- A (10 x 2 = 20 Marks)
(Answer all Questions)

Q. No	Questions	Marks	CO	BL
1	What is the linearity principle in differential equations?	2	1	L2
2	Find the particular integral of $(D^2-2D+1)y = xe^x sinx$.	2	1	L1
3	Find the Laplace transform of unit step function.	2	2	L2
4	Find $L^{-1}\left[\frac{1}{\sqrt{s+2}}\right]$	2	2	L1
5	If the Fourier series expansion of $f(x)=\begin{cases}0, & -\pi<x<0\\ \frac{\pi x}{4}, & 0<x<\pi\end{cases}$ is $\frac{\pi^2}{16}-\frac{1}{2}\sum_{n=1,3,5,..}^{\infty}\frac{cosnx}{n^2}+\frac{\pi}{4}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}sinnx$, then find the value of the series $\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}$.	2	3	L2
6	Find the Root mean square value of $f(x)=x-x^2$ in $-1<x<1$.	2	3	L2

7	Express the function $f(x) = \begin{cases} 1, & \lvert x \rvert \le 1 \\ 0, & \lvert x \rvert > 1 \end{cases}$ as a Fourier integral.	2	4	L1
8	If $f(x)$ is an even function of x, then prove that $F(s) = F[f(x)]$ is also an even function of s.	2	4	L2
9	Find the Z-transform of the sequence $\{4, 8, 16, 32, \ldots \ldots\}$	2	5	L1
10	Form a difference equation by eliminating the arbitrary constant A from $y_n = A3^n$.	2	5	L2

PART- B (5 x 13 = 65 Marks)

(Restrict to a maximum of 2 subdivisions)

Q. No	Questions	Marks	CO	BL
11 (a) (i)	Solve $(D^2 + 4)y = \sec 2x$ by method of variation of parameters.	7	1	L3
(ii)	Solve $(x+2)^2 \frac{d^2y}{dx^2} - (x+2)\frac{dy}{dx} + y = 3x + 4$.	6	1	L3
	(OR)			
11 (b) (i)	Solve $(D^2 - 2D - 3)y = 2e^x - 10\sin x$ by method of undetermined coefficients.	7	1	L3
(ii)	Solve: $\frac{d^2y}{dt^2} = x$ and $\frac{d^2x}{dt^2} = y$.	6	1	L3
12 (a) (i)	Find $L\left[e^{-t}\int_0^t t\cos t\,dt\right]$ & $L^{-1}\left[\frac{s+3}{s^2-4s+13}\right]$.	3+3	2	L4
(ii)	Verify the initial and final value theorem for the function $f(t) = 1 + e^{-t}(\sin t + \cos t)$	7	2	L4
	(OR)			
12 (b) (i)	Find the Laplace transform of the periodic function $f(t) = \begin{cases} 1 + \frac{(t-a)}{a}, & 0 < t < a \\ 1 - \frac{(t-a)}{a}, & a < t < 2a \end{cases}$ with period $2a$.	6	2	L4
(ii)	Using convolution theorem, find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$	7	2	L4

13 (a) (i)	Obtain the Fourier series for the function $f(x) = 1 - \frac{2}{l}\lvert x\rvert$ in $-l \le x \le l$ and hence find the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots + \infty$.	7	3	L3
(ii)	Find the half range sine series for the function $f(x) = \begin{cases} x; & 0 < x < \frac{\pi}{2} \\ \pi - x; & \frac{\pi}{2} < x < \pi \end{cases}$	6	3	L3
	(OR)			
13 (b) (i)	Obtain the Fourier series expansion for the function $f(x) = \begin{cases} 1; & 0 < x < 1 \\ 2; & 1 < x < 2 \end{cases}$	6	3	L3
(ii)	Find the half range cosine series for the function $f(x) = x(\pi - x)$ in $0 < x < \pi$. Hence find the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$.	7	3	L3
14 (a) (i)	Find the Fourier transform of $f(x) = \begin{cases} x; & \lvert x\rvert \le a \\ 0; & \lvert x\rvert > a \end{cases}$ and hence find the integral $\int_0^{\infty} \frac{sinx - xcosx}{x^2} sin\left(\frac{x}{2}\right) dx$.	7	4	L4
(ii)	Obtain the Fourier sine transform of e^{-ax} and using Parseval's identity find the integral $\int_0^{\infty} \frac{x^2}{(x^2+a^2)^2} dx$.	6	4	L4
	(OR)			
14 (b) (i)	Find the Fourier transform of $f(x) = \begin{cases} \frac{\sqrt{2\pi}}{2a}; & for \lvert x\rvert \le a \\ 0; & for \lvert x\rvert > a \end{cases}$ and hence evaluate $\int_0^{\infty} \frac{sinx}{x} dx$ & $\int_0^{\infty} \left(\frac{sinx}{x}\right)^2 dx$.	7	4	L4
(ii)	If the Fourier cosine transform $f(x)$ is $\begin{cases} \frac{1}{2\pi}\left(a - \frac{s}{2}\right); & if\ s < 2a \\ 0; & if\ s \ge a \end{cases}$, then find its function $f(x)$.	6	4	L4

15 (a) (i)	Determine the Z- transforms of $r^n cosn\theta$ & $r^n sinn\theta$	5	5	L3
(ii)	Find the inverse Z-transform of $\frac{z^3+3z}{(z-1)^2(z^2+1)}$ using partial fractions.	8	5	L3
	(OR)			
15(b) (i)	Verify convolution theorem for $u_n = n$ & $v_n = 1$.	5	5	L3
(ii)	Solve the equation $y(n+2) + 4y(n+1) + 3y(n) = 3^n$ with $y(0) = 0,\quad y(1) = 1$ by using Z - transform.	8	5	L3

PART- C (1 x 15 = 15 Marks)

(Q.No.16 is compulsory)

Q. No	Questions	Marks	CO	BL
16 (i)	Solve $y'' - 3y' + 2y = 4t$, given $y(0) = 1$ & $y'(0) = -1$ by using Laplace transform.	8	2	L5
(ii)	Compute the first three harmonics of the Fourier series of $f(x)$ from the following data.	7	3	L5

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	10	14	19	17	15	12	10

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